Network-based characterization of extreme events in time series

Juliane T. Moraes[1], Lucas Lacasa[2], and Cristina Masoller[1]

[1]Universitat Politècnica de Catalunya, Departament de Fisica, Rambla Sant Nebridi 22, Terrassa 08222, Barcelona, Spain.

[2]Institute for Cross-Disciplinary Physics and Complex Systems (CSIC-UIB), Campus UIB, Palma de Mallorca 07122, Spain.

(juliane.teixeira@upc.edu)

Introduction

Extreme events play a central role in many complex systems, from climate dynamics to brain activity. Beyond amplitude-based definitions, the visibility graph approach captures structural properties of the time series, revealing dynamical features associated with extreme events that may be not accessible through standard statistical methods.

Methods

Visibility Graph (VG):

The visibility graph method consists of mapping a time series onto a network structure through the following criteria. Given two values (t_a, y_a) and (t_b, y_b) will have visibility, i.e., will be connected in the associated graph, if any point (t_c, y_c) between them satisfies [1]:

$$y_c < y_b + (y_a - y_b)\frac{(t_b - t_c)}{(t_b - t_a)}. \tag{1}$$

A schematic illustration of the visibility criteria in given in Fig. 1.

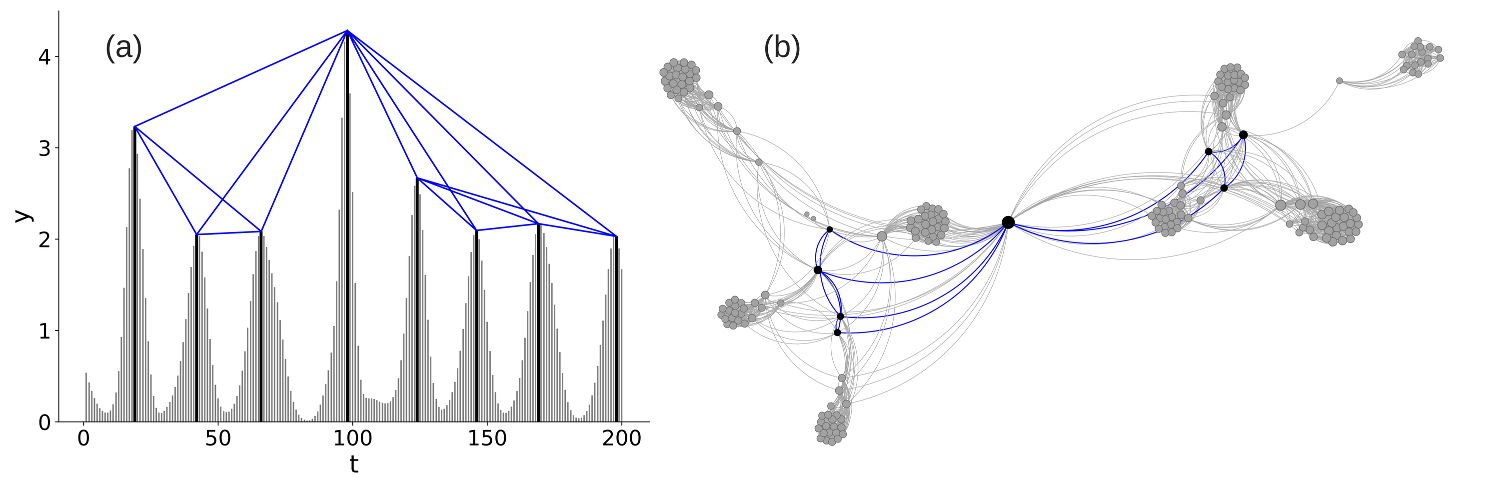

Figure 1: Visibility graph (b) constructed from the peak time series shown in (a) (black bars). The underlying gray time series in panel (a) is obtained from a deterministic rogue wave model [3]. Blue lines in (a) illustrate the visibility criterion Eq. (1) and correspond to the edges in the visibility graph in (b).

Ranking the nodes in the VG:

Since each node i in the VG represents a value y_i in the time series, we characterize the VG using centrality measures, for example, the degree k_i, i.e., the number of connections of a node [2]. To find and classify the most relevant nodes, we rank them first by their corresponding height in the time series and also by their degree in the VG. An example using the VG from the time series in Fig. 2 is presented in Tab. 1.

Table 1: VG ranking method displaying the first six positions in the ranks by height and degree.

(a) Ordering by height

rank by height	rank by degree	node	height	degree
1	1	**1201**	10.5	221
2	4	1200	8.3	104
3	6	1202	7.7	77
4	2	298	6.5	138
5	3	1378	6.1	119
6	7	297	6.0	71

(b) Ordering by degree

rank by height	rank by degree	node	height	degree
1	1	**1201**	10.5	221
4	2	298	6.5	138
5	3	1378	6.1	119
2	4	1200	8.3	104
27	5	553	4.0	88
10	5	930	4.7	88
3	6	1202	7.7	77

We applied this framework to a time series generated from a simple rate equation model, which shows good qualitative agreement with experiments on optically injected semiconductor lasers [3]. This model generates time series as the one shown in Fig. 2 and may exhibit extreme amplitude events.

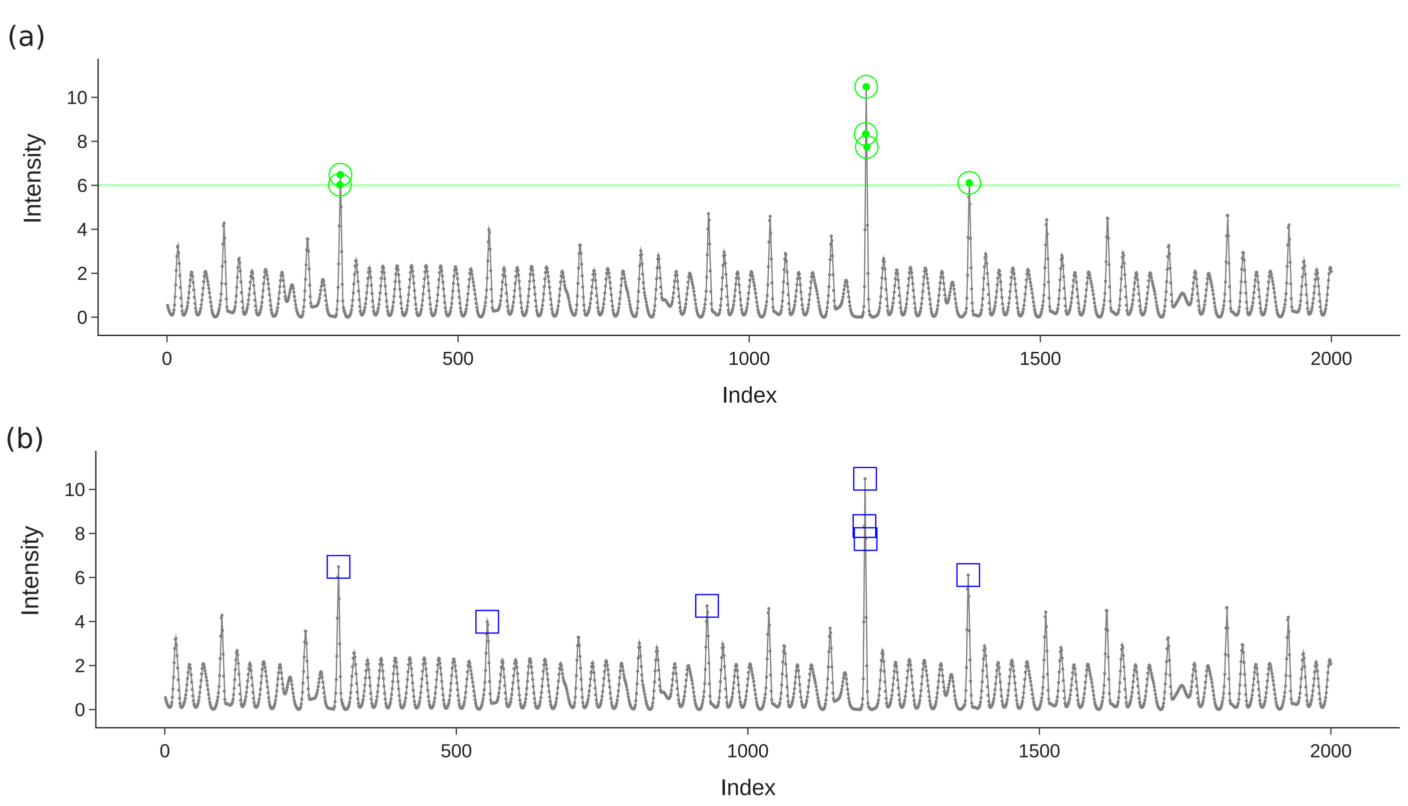

Figure 2: Time series of the laser intensity obtained by the deterministic rogue wave model [3]. Panels (a) and (b) show the extreme events identified by the height and degree of the nodes in the generated VG, respectively.

The points identified by height in Fig. 2(a) can be reproduced using a threshold criterion and correspond to Tab. 1(a). In contrast, the degree-based selection (Tab. 1(b)), shown in Fig. 2(b), reveals additional points with local importance not captured by the height criterion.

Results and Discussion

In the following, we show results applying the method for a sub-sample of the time series composed of only the peak values. It is possible to observe that the extreme events are displayed in the first positions of the ranks in Fig. 3 for this case.

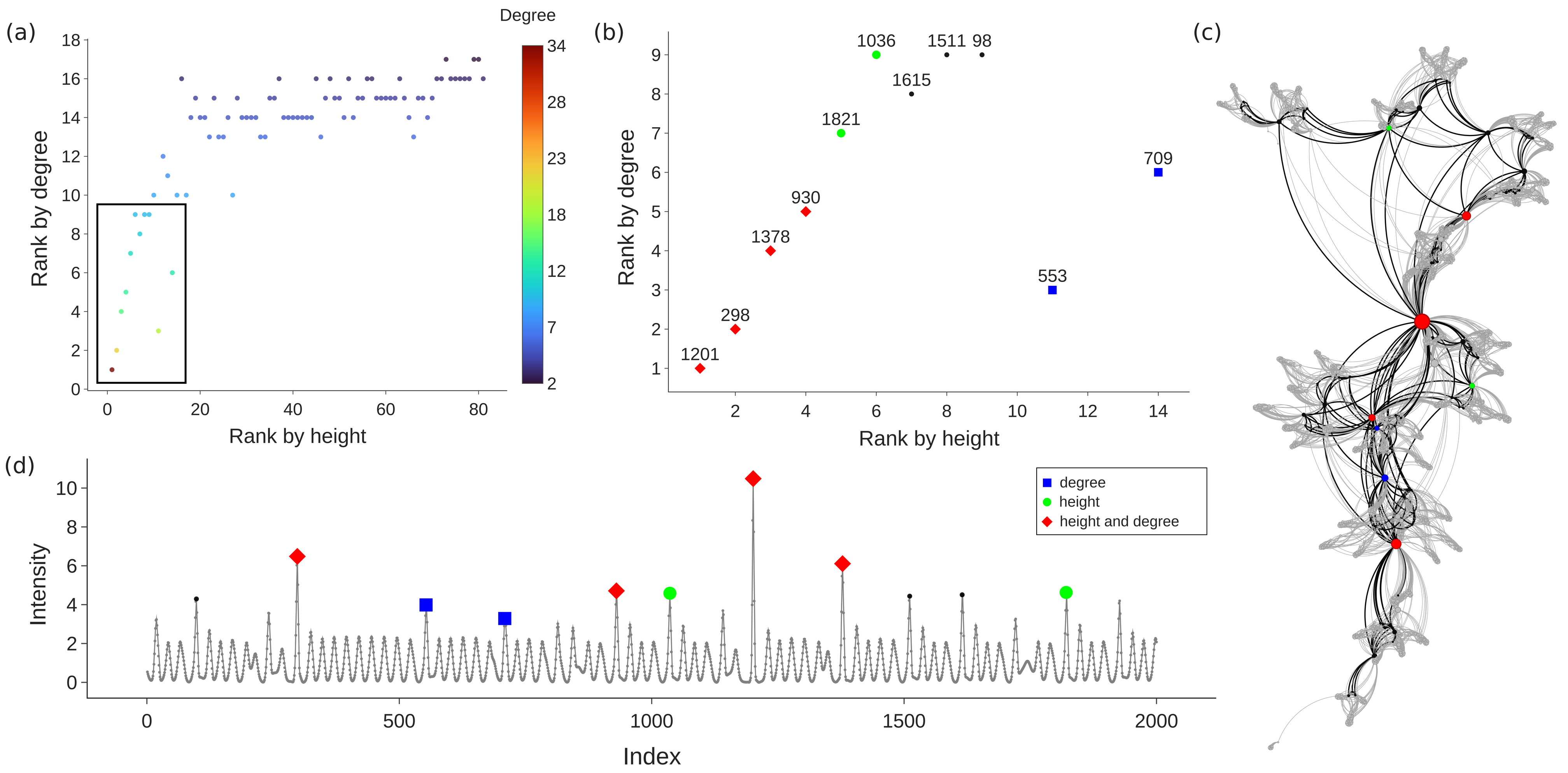

Figure 3: VG ranking method applied to a time series generated by a deterministic rogue wave model [3]. Panel (a) shows the ranking by degree and height. The highlighted region in (a) is zoomed in panel (b), where points identified by the degree, the height, and by both can be observed. These points are also marked in the time series shown in panel (d). Panel (c) displays the visibility graph constructed from the full time series and from the peaks; nodes marked in red, blue, and green correspond to the identified extreme events in panels (b) and (d).

We now apply the same framework to real data from the Niño3.4 index anomaly [4]. In this case, there are no pronounced extreme events as in the previous example. Nevertheless, the method successfully identifies points in the time series corresponding to El Niño episodes, as it is shown in Fig. 4.

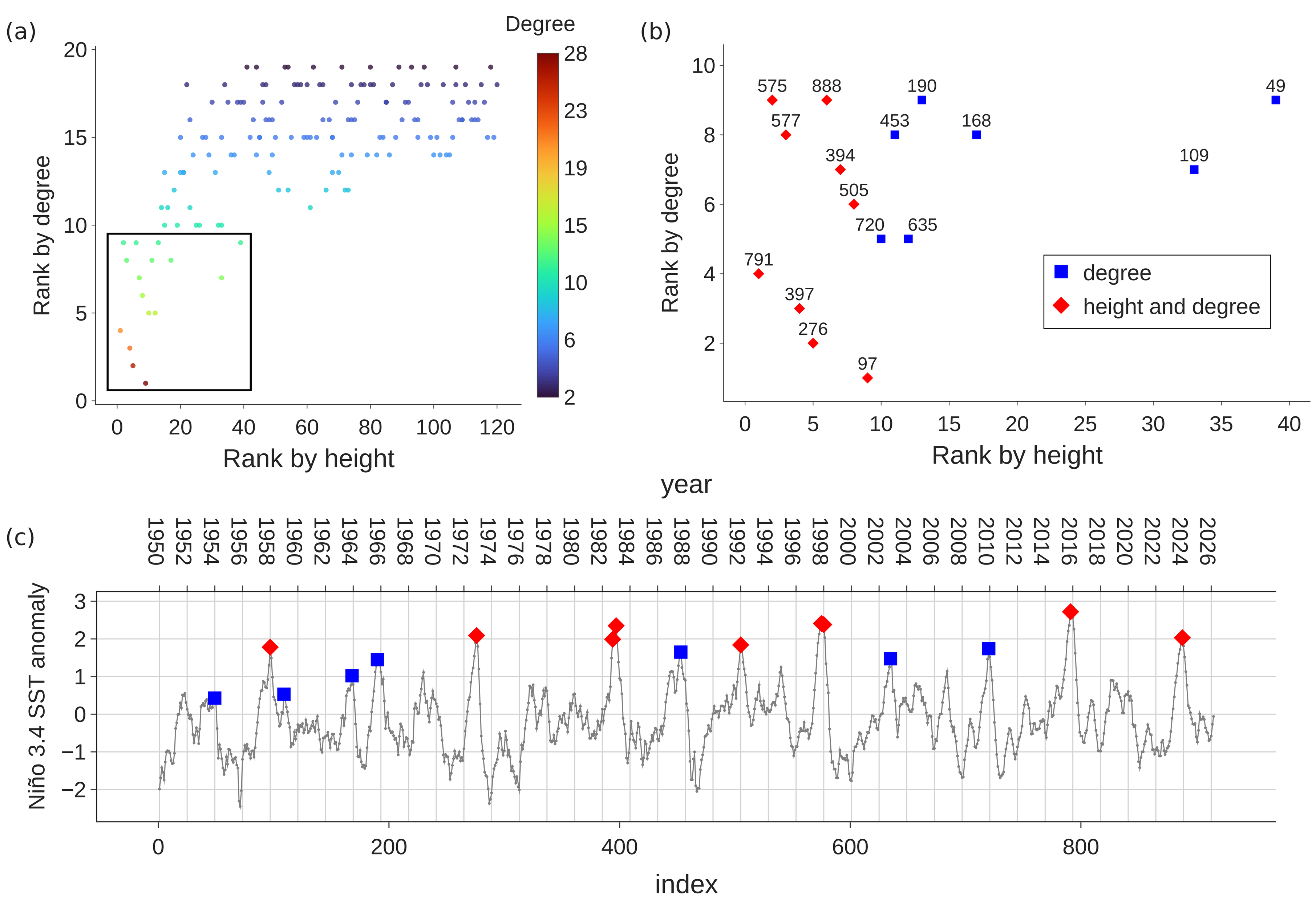

Figure 4: VG ranking method applied to the Niño3.4 index anomaly time series [4]. Panel (a) shows the ranking by degree and height. The region highlighted in (a) is zoomed in panel (b), where points identified by the degree criterion and by both criteria can be observed. These points are also marked in the time series shown in panel (c).

Conclusions

Our analysis indicates that extreme events identified through visibility graphs degree sequence are not solely determined by their amplitude, but also by their relative location on the temporal organization of the series. This visibility-based analysis allows us to obtain a ranking of the data values in a time series that are locally extreme. The method is parameter-free and can be useful for non-stationary time series.

Acknowledgments

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References

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[4] NOAA Physical Sciences Laboratory (PSL), based on NOAA Climate Prediction Center (CPC) Niño 3.4 Index. Available at: `https://psl.noaa.gov/data/timeseries/month/DS/Nino34_CPC/`(accessed 29 April 2026).